Dynamics of Cell Assemblies in Binary Neuronal Networks

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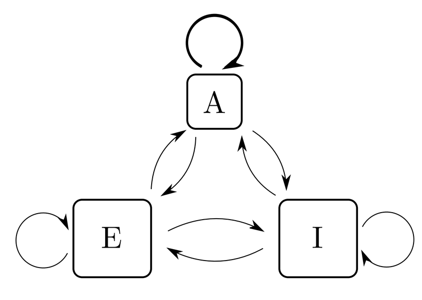
Motivation

- Gain an analytic understanding of cell assembly dynamics
- Search for biologically plausible size, clustering strength, etc.
- Find effective equation of assembly in response to external input
- Could cortical assemblies explain the sharp phase locking of unitary events to LFP oscillations described by [1] ?
- Mean field theory is expected to break down in the bistable regime, thus try to calculate systematic fluctuation corrections using a path-integral formulation.

Model

- Sparse random E-I network with 2000-10000 neurons per population in the balanced state as a background
- Introduce a single assembly by taking N_A ($\sim$ 40-150) neurons from the exitatiory population and increasing the connection probability between them by **factor a** while keeping the total number of inputs constant.
- Using binary neurons with a Heaviside gain function for analytic tractability
- Excitatory synapses have strength $j_E \sim \frac{1}{\sqrt{N}}$, inhibitory $j_I = -g j_E$
- Simulations are done with NEST [2]



Model consisting of assembly in excitatory and inhibitory bath.

Mean Field Theory vs. Simulation

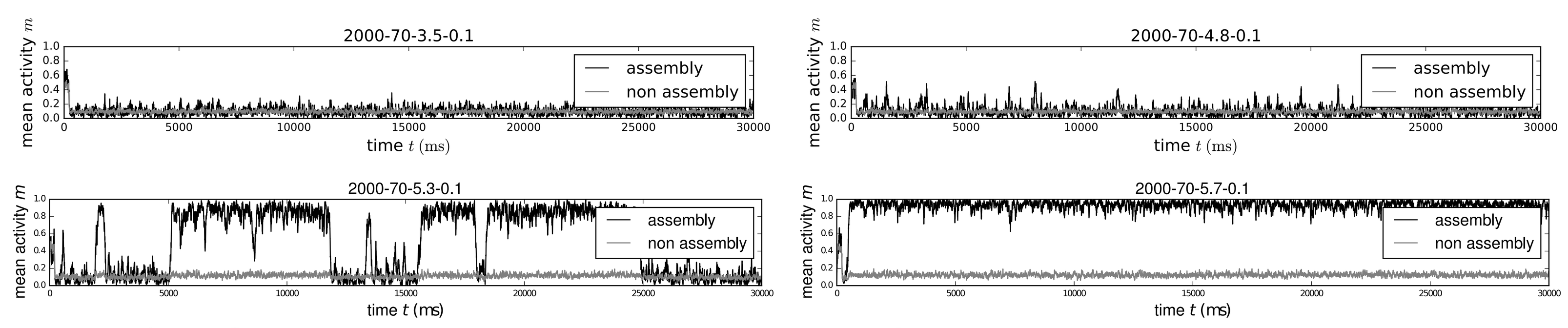


Figure 1. Simulated activity in the model network for **a**= 3.5, 4.8, 5.3 and 5.7. For **a**=5.3, fluctuation driven transitions between two stable states are observed.

The mean field equation with $\alpha \in \{E, I, A\}$ [3] [4]:

$$\tau \frac{\partial}{\partial t} m_\alpha + m_\alpha = \frac{1}{2} \text{erfc}\left(\frac{\theta - \mu_\alpha(\boldsymbol{m})}{\sqrt{2}\sigma_\alpha(\boldsymbol{\mu})}\right). \quad (1)$$

It is derived from the exact Master eq. by assuming the summed inputs to a neuron to be Gaussian distributed as $\mathcal{N}(\mu_k, \sigma_k)$ and vanishing cross-covariances.

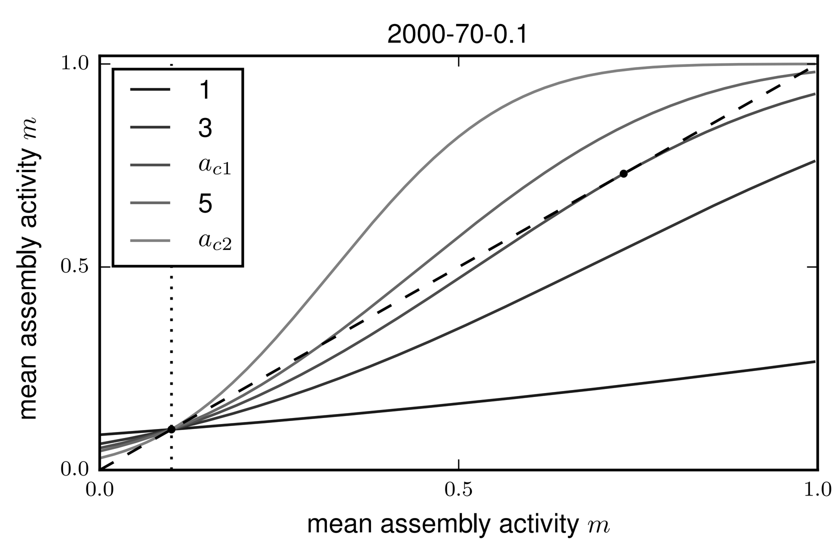
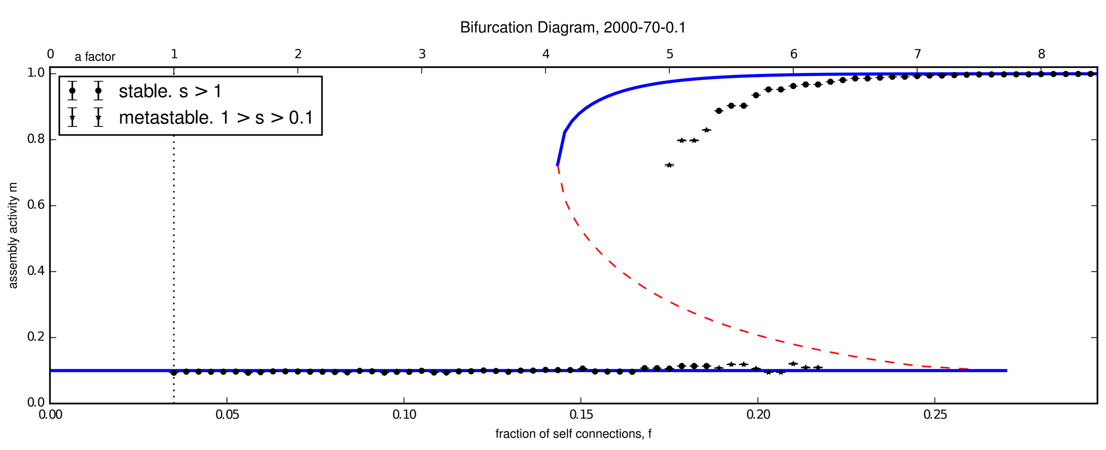


Figure 2. Graphical representation of (1) for the assembly model (N=2000, N_A=70, m=0.1) using various clustering strengths **a**. Intersections between identity and sigmoidal effective activation function give the solutions to the stationary equation. Depending on **a**, groundstate, ground- and up-state, or only the up-state are stable. Transitions between regimes are given by the critical points $\boldsymbol{a}_{c1}$ and $\boldsymbol{a}_{c2}$. The up-state cannot have activity smaller 0.5 .

Figure 3. Bifurcation diagram showing the effect of increased clustering on the stable solutions. Blue lines: prediction from MFT, dots: simulation results. Due to large fluctuations, the approximation is inaccurate in the vicinity of the critical points.



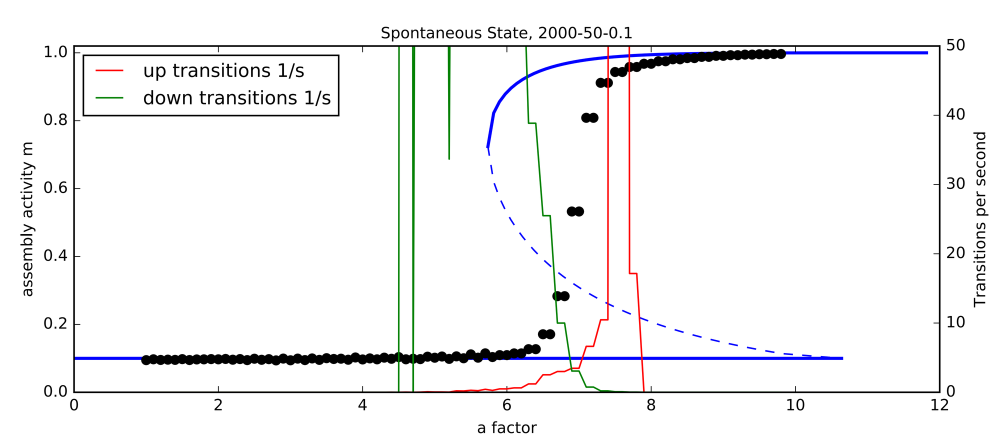


Figure 4. MFT bifurcation diagram overlaid with the simulated spontaneous activity and transition rates.

Field Theory

- Potent mathematical tool used in many-particle physics and quantum field theory [6,7,8,9].
- The system is fully described by an action $S[x(t,r), \tilde{x}(t,r)]$
- Probability distribution in state space is $\exp(S[x, \tilde{x}])$
- Fourier transform: "moment generating function"

$$\mathcal{Z}[j, \tilde{j}] = \int \mathcal{D}x\mathcal{D}\tilde{x} \, \exp\left(S(x, \tilde{x}) + jx + \tilde{j}\tilde{x}\right)$$

- e.g. $\langle xx \rangle = \frac{1}{\mathcal{Z}} \frac{\partial^2}{\partial j^2} \mathcal{Z}[j, \tilde{j}]\Big|_{j,\tilde{j}=0}$, also: $\mathcal{Z}[0,0]$ corresponds to partition function. "Path-" or "functional integral" due to integration over all points of $x(t,r)$.
- Cumulant generating function containing only connected graphs

$$W[j, \tilde{j}] = \ln \mathcal{Z}[j, \tilde{j}].$$

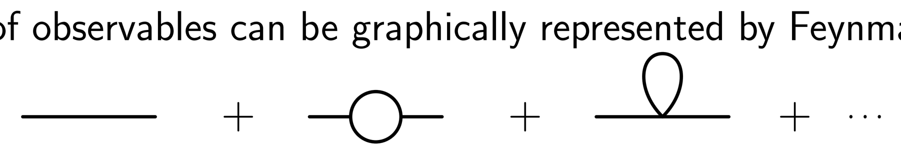
- By Legendre transform, the effective Potential is obtained

$$\Gamma[x^*, \tilde{x}^*] = \sup_{j,\tilde{j}} jx^* + \tilde{j}\tilde{x}^* - W[j, \tilde{j}].$$

- Typically, action consists of quadratic part and higher order interaction terms: (where often l is large, and/or ϵ is small)

$$S[x, \tilde{x}] = l\left(\mathbf{x}^T \boldsymbol{A} \mathbf{x} + \epsilon V[x, \tilde{x}]\right)$$

- Expansions of observables can be graphically represented by Feynman diagrams, eg.



- Allows systematic fluctuation corrections by loopwise expansion and the use of techniques like renormalization, developed over decades in theoretical physics.

Application to Binary Neurons

- Randomness of connectivtiy can be averaged over (self averaging property), incorporating the variability between connectivity realizations into the description, leading to homogeneously connected populations coupled to fluctuating background fields (Disorder average). A pedagogical introduction to the method is given in [9].
- Hubbard-Stratonovich transform in these fields leads to path-integral with an action of the form

$$S[R_{1,2}, Q_{1,2}] = R_1 R_2 + Q_1 Q_2 + \ln \Omega[R_{1,2}, Q_{1,2}]$$

where $R_1(t) = \frac{\bar{g}}{\sqrt{N}} \sum_j x_j(t)$ is the mean input and $Q_1(t,s) = \frac{g^2}{N} \sum_j x_j(t)\, x_j(s)$ the input variance due to autocorrelation, along with their auxiliary fields R_2 and Q_2, acting as source terms.

- In zero loop order, ie. no fluctuations, we have $\Gamma_0 = -S$ and the equation of state

$$\frac{\partial \Gamma_0}{\partial\{R_i, Q_i\}}\Big|_{\{R_i^*, Q_i^*\}} = 0$$

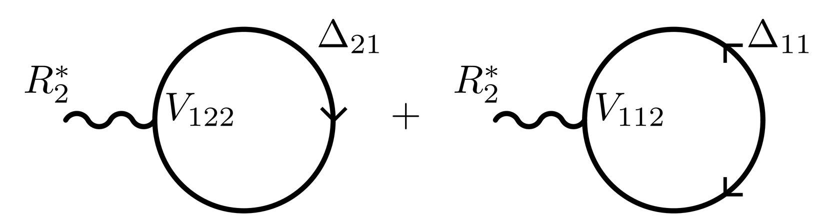
leads to Gaussian integrals, giving eventually

$$\tau \frac{\partial}{\partial t} m(t) + m(t) = \frac{1}{2} \text{erfc}\left(\frac{\theta - R_1^*(t)}{\sqrt{2Q_1^*(t,t)}}\right), \quad (2)$$

and $Q_1^*(t,t) = J_0^2 p(1-p)\, m(t)$.

This agrees with the naive mean field solution, as $\mu(t) = R_1^*(t)$ and $\sigma^2(t) = Q_1^*(t,t)$.

- To calculate fluctuation corrections to leading order, the 1-loop diagrams are included. There are only two such diagrams contributing to the mean input $R_1^{*(1)}$:



- The propagators are calculated in frequency domain, e.g.

$$\Delta_{21}(\omega) = 1 + \frac{k(R_1^*)}{1 - k(R_1^*) + i\tau\omega}$$

where $k(R_1^*)$ is proportional to the neurons' susceptibility, and Δ_{21} can be interpreted as the response of the mean activity to external input.

Outlook

- Extend model generality, eg. clustering by synapse strength, comparison to spiking assembly
- Effective eq. for assembly response to external input
- Systematic corrections to MFT
- Interaction in network of many overlapping assemblies
- Calculate transition rates via Arrhenius factor
- Find way to capture the bistable regime in the path-integral expansion.

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